A MECHANISTIC STRAIN ENERGY FUNCTION FOR THE HUMAN ANNULUS FIBROSUS

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The complex biomechanical function of the annulus fibrosus, a component of the intervertebral disk, is facilitated by a composite architecture of collagen fibers embedded in a ground substance that is primarily composed of proteoglycans. This structural organization is similar to an angle-ply composite and leads to tremendously anisotropic material properties. A composite continuum theory that was first proposed by Spencer [1] has been previously applied to the annulus by several researchers [2-5]. This theory provides a mathematical structure that reflects the composite tissue architecture; however the previous models do not explicitly represent specific tissue features, such as collagen crosslinking, that may be important in understanding the effects of aging and degeneration. The objectives of this study were to validate an annular strain energy function with individual terms to represent specific tissue features, and to demonstrate that the mechanical changes due to increased crosslinking can be modeled by increasing particular material coefficients of the strain energy function.

The proposed strain energy function for the annulus consists of a sum of separate terms:

$$W = a_1(I_3 - 1/I_3)^2 + a_2(I_1 I_3^{-1/3} - 3)^2 + \frac{a_3}{b_3}\left(e^{b_3(I_9-2)} - b_3 I_9\right) + \frac{a_4}{b_4} e^{b4(I_{11} - I_9^2 + 2I_{10})} + \frac{a_5}{b_5}\left(e^{b_5(J_9-2)} - b_5 J_9\right)$$

where the 1st and 2nd terms represent the response of the matrix, the 3rd term models the collagen fibers and the 4th and 5th terms represent the response of the collagen crosslinks. Using previously described methods, values of the strain energy coefficients {a_1, a_2, a_3, b_3, a_4, b_4, a_5, b_5} were determined by conducting a non-linear regression to the mean elastic stress-strain response from wide range of experimental protocols. Another non-linear regression was then conducted to just the mean axial stress-strain data from tissue that had been artificially crosslinked, allowing only the coefficients a_4 and a_5 to vary and setting the rest of the coefficients equal to the previously determined values.

The best fit values for the strain energy coefficients were {0.0876, 0.00415, 0.00378, 32.2, 0.0237, 0.0019, 0.000226, 7.48}. These coefficients resulted in stress-strain curves that lie within one standard deviation for all experimental deformations. Additionally, we found an excellent fit to the axial tension data from the crosslinked tissue with values of $a_4 = 0.0456$ and $a_5 = 0.000408$ (increases of 93% and 81% over above values, respectively) and all other coefficients as listed above.

The proposed strain energy function, with separate terms to represent the contributions of the annular constituents, accurately captures the mean elastic response to multiple experimental deformations. Additionally, the coefficients a_4 and a_5 are shown to correlate to crosslink density. This is a first step towards developing a mechanistic constitutive relationship that relates specific features of tissue architecture to material properties. If successful, this approach may be used in the future to elucidate the pathomechanics of aging and degeneration in the intervertebral disc.

References

[1] Spencer AJM, Continuum theory of the mechanics of fibre-reinforced composites, Spernger-Verlag, New York, 1984 [2] Wu H and Yao R, J Biomech, 1976 [3] Klisch SM and Lotz JC, J Biomech, 1999 [4] Elliott DM and Setton LA, J Biomech Eng 2000 [5] Eberlein R et al,, Compt Meth Biomech Biomed Eng 2001