# VOLUMETRIC GROWTH OF ELASTIC MATERIALS AND FINITE VOLTERRA DISLOCATIONS

JAMES CASEY* AND GÉRARD A. MAUGIN†

* Departments of Mechanical Engineering and Bioengineering,
University of California, Berkeley.
jcasey@me.berkeley.edu

† Laboratoire de Modélisation en Mécanique,
Université Pierre et Marie Curie, Paris.

When volumetric growth or resorption occurs in biological materials, residual stresses are generally induced [1-3]. Relief of these stresses by local unloading would lead to a collection of local configuratons that would not fit together to form a global stress-free configuration of the material, i.e., the compatibility conditions of continuum mechanics would not be satisfied. In some important cases, however, the residual stresses resulting from growth of multiply connected elastic bodies can be relieved by making cuts, yielding a global stress-free configuration. In such cases, the theory of finite Volterra dislocations [4] becomes relevant. A Volterra dislocation of a continuum is a deformation which has a jump discontinuity in displacement across some singular surface, but which possesses a finite strain field that is continuous and also has continuous second partial derivatives. In the context of linear elasticity, Weingarten [5] was the first to realize the physical importance of such discontinuous solutions and Volterra soon developed the subject in a series of papers (see [6]). The main result of the classical theory is Weingarten's theorem : In an infinitesimal Volterra dislocation of a continuum, the cut faces are necessarily related to one another by an infinitesimal rigid displacement. Weingarten's theorem also holds for finite deformations [4] (but its proof is entirely different from the infinitesimal case).

We present a theory for volumetric growth of elastic materials via finite Volterra dislocations. Among the examples considered is that of a doubly connected elastic body which grows naturally from some stress-free undeformed configuration into a configuration from which a single Volterra cut can relieve it of residual stresses. The simply connected intermediate stress-free configuration is then related to the undeformed configuration by another Volterra dislocation. Both of these Volterra dislocations have the same Burgers vector and the same Frank tensor.

**References**

[1] E. K. Rodriguez, A.Hoger, and A. D. McCullogh, "Stress-dependent finite growth in soft elastic tissues," *J. Biomechanics* **27**, 455-467, 1994.
[2] R. Skalak, S. Zargaryan, R.K. Jain, P.A. Netti, and A. Hoger, "Compatibility and the genesis of residual stress by volumetric growth," *J. Math. Biol.* **34**, 889-914, 1996.
[3] S. M. Klisch, T.J. Van Dyke, and A. Hoger, "A theory of volumetric growth for compressible elastic materials," *Math. Mech. Solids* **6**, 551-575, 2001.
[4] J. Casey, "On Volterra dislocations of finitely deforming continua," *Math. Mech. Solids* **9**, 473-492, 2004.
[5] G. Weingarten, "Sulle superficie di discontinuità nella teoria elasticità dei corpi solidi," *Atti Accad. Lincei Rend.* Series **5**, **10**, 57-60, 1901.
[6] V. Volterra and E. Volterra, "Sur les Distortions des Corps Élastiques (Théorie et Applications)," *Mémorial des Sciences Math.* Fascicule CXLVI, Gauthier-Villars, Paris, 1960.

**Keywords:** biomechanics, growth, Volterra dislocations