SCALE AND BOUNDARY CONDITIONS EFFECTS ON ELASTIC MODULI OF TRABECULAR BONE

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We study scale and boundary conditions effects on elastic moduli of trabecular bone. The trabecular bone structure is very complex due to its randomness and spatial heterogeneity. To simplify the analysis we choose to represent bone as having an idealized periodic structure. More specifically, we model bone as having periodic prismatic structure, either two- or three-dimensional. In this paper we are interested in predicting the apparent elastic moduli of such an idealized geometric model.

If the "region of observation" (window) is smaller than the Representative Volume Element then the moduli depend on size of that region and boundary conditions and they are referred to as the apparent moduli [1]. In order to investigate the effect of scale and boundary conditions on elastic moduli we vary the size of the "window" and we apply several different boundary conditions: displacement, traction, periodic and mixed boundary conditions. The analysis using periodic boundary conditions gives the effective response while the remaining three boundary conditions give the apparent moduli. The results obtained using our mixed boundary conditions are very close to the effective ones. The apparent moduli calculated using displacement boundary conditions bound effective moduli from above while the moduli obtained using traction boundary condition results bound the effective moduli from below. The larger is the size of the window the closer are the bounds. Since our models are effectively orthotropic with cubic symmetry we only apply three loadings: unidirectional, hydrostatic and shear loadings for our two- and three-dimensional geometries.

We conduct our analysis using a finite element method (ANSYS). We investigate the effect of mesh size, the mismatch in moduli (elastic modulus of bone tissue versus bone marrow), the size of the window, the effects of bone structures (sharp edges versus rounded ones, square pores versus circular ones), and boundary conditions on the elastic moduli of the idealized geometric models of trabecular bone.

In addition, we are interested in the fundamental issues and check for the satisfaction of the average strain or stress theorems and the satisfaction of the Hill condition, which guarantees that the results obtained using direct approach or energy approach are the same. We compare the results of the case when bone marrow (fill) has very small but non-zero moduli with the case in which only the bone tissue (representing bone marrow as void) is modeled.

This study can give guidance in determining the sufficient size of bone samples used in experiments or computations so the effect of boundary conditions is minimized. These results are also applicable for other porous/cellular materials.

Reference

[1] C. Huet, "Application of Variational Concepts to Size Effects in Elastic Heterogeneous Bodies," *J. Mech. Phys. Solids* **38**, 813-841, 1990.

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