GREEN'S FUNCTIONS FOR REGIONS WITN INCLUSIONS

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It is hard to overestimate the crucial role that Green's functions play in the qualitative analysis of boundary value problems for either ordinary or partial differential equations. It has also been discovered, in recent years [1-5], that Green's function-based methods appear to possess promising computational potential (impressive convergence rate and high accuracy level), in solving many classes of applied boundary value problems in engineering and science. This feature is especially welcome for optimization or other nonlinear problems, most of which (if not all) are usually solved on a time consuming basis of successive approximations.

There exists however a notable constraint that yet refrains the engineering community from a wide use of the Green's function-based methods in practice. This is a lack of such representations of required Green's functions, which are either convenient for computer implementations in their current form or can readily be adjusted for this purpose. Clearly, presence of singular components in Green's functions is behind of their computability.

In [2] one can find a vast number of Green's functions constructed for a variety of boundary value problems for Laplace, Klein-Gordon, biharmonic and other applied PDEs. Many of those were obtained for the first time, while others were found in a form that can notably be better adapted for computer implementations compared to their forms available in the existing literature. Since we managed to obtain singular components of Green's functions in analytic form, our representations are always computer-friendly.

To make the notion of Green's function applicable to a specific type of media filled in with piecewise homogeneous materials, we have introduced [3] the notion of *matrix of Green's type*. In the present study, a further extension of our approach is provided where a number of particular matrices of Green's type have been obtained for Laplace and Klein-Gordon equations so that they are applicable to regions of nonstandard form filled in with isotropic homogeneous materials containing foreign inclusions.

References

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