THE STABILITY OF SELF-GRAVITATING ELASTIC BODIES.

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The classic studies of Lame and Thompson in the mid nineteenth century on the deformation of an elastic sphere subject to general body forces provided the starting point for investigations of planetary bodies under gravitational forces . A topic of particular interest was that of tidal deformation, namely the overall distortion of a planet due to its gravitational interactions with another body. A famous study of this problem by A.E.H. Love in 1911 [1] studied the tidal deformation of a homogeneously dense planetary body due to the gravitational effect of a neighboring mass referred to as the tide raiser. The stability of a self-gravitating elastic body could not be studied due to the limitation of classical (infinitesimal) elasticity theory.

In this talk, the classical problem of tidal deformations in planets is revisited by using exact elasticity theory to study the problem of self-gravitational collapse of a uniformly dense, compressible sphere [2]. Material parameter values are found for which non-radial perturbations added to the pre-stressed state of a self-gravitationally collapsed planet can grow indicating the possibility of tidal instabilities. Planets with Earth-like properties are found to be stable in this regard, and the stability analysis suggests a criterion for the limiting properties of planets forming from an accretion disc.

References

[1] A.E.H. Love, "The gravitational stability of the Earth", *Philosophical Transactions of the Royal Society of London. Series A, Containing Papers of a Mathematical or Physical Character*, **207**:171241, 1908.
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