DISCONTINUOUS GALERKIN METHODS WITH PLANE WAVES FOR TIME-HARMONIC PROBLEMS

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The practical motivation of this work is the numerical simulation of noise radiation from turbofan engines. This involves solutions of the linearized Euler equations which describe the propagation of acoustic and hydrodynamic linear disturbances on a strongly non-uniform mean flow. In addition, these applications generally require to solve large three-dimensional models at high frequencies.

To solve time-harmonic wave propagation problems, several numerical techniques have been recently developed using local solutions of the problem at hand to build the shape functions. The Partition of Unity method [1], the Discontinuous Enrichment method [2], the Ultra Weak Variational Formulation [3] and the Green's function discretization scheme [4], among others, belong to this category of 'physics-based' numerical methods.

This communication reports the development of a discontinuous Galerkin method, with numerical flux, using local plane-wave solutions to approximate the problem inside each element. The method is devised for a general system of conservation equations and examples of applications are presented for the linearized Euler equations. The plane-wave basis is given by a dispersion analysis of the continuous problem with locally uniform coefficients and contains two separate sets of acoustic and hydrodynamic waves. It is found that an upwind numerical flux is particularly interesting in the context of plane-wave basis.

Results show that this method is very accurate and yields a significant improvement from standard discontinuous Galerkin methods using polynomial basis. However, this method also presents the same conditioning problem observed with other similar methods but for typical applications the conditioning remains acceptable. In fact, the use of iterative methods to solve the resulting algebraic problem is also investigated since direct solvers can be prohibitively expensive for large problems.

The connection with others physics-based numerical is discussed.

References

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