BOUNDS AND ESTIMATES ON ELASTIC PROPERTIES AND WAVE SPEEDS FOR ROCKS WITH DRY OR SATURATED CRACKS

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The elastic properties of rocks, concrete, and other brittle or quasi-brittle materials may be dominated in the dry state by the behavior of very flat cracks, containing very little or no porosity. The majority of work on effective medium theories (both estimates and bounds) has concentrated instead on systems in which the elastic behavior depends on the volume fractions of the constituents. But the flat crack limit clearly needs separate treatment. The earlier work on this subject has been reviewed by Laws and Dvorak [1] and by Zimmerman [2]. There remain controversies regarding use of the differential and self-consistent schemes for property estimation in these systems, and these controversies are known and well-documented.

Sayers and Kachanov [3] show that the anisotropic behavior of cracked systems (at least with small crack densities) can be determined easily by first computing effective properties for the isotropic case using some reliable method (they show that the differential scheme is one good choice). Then the elastic potential is linearized and two coefficients can then be determined from the isotropic results. Having these two coefficients is sufficient information to determine the elastic properties for any other desired (anisotropic) symmetry, as long as the crack density remains sufficiently small.

The present work will apply another type of symmetric self-consistent scheme called the CPA (coherent potential approximation) to the crack problem, and then compare and contrast the results to those obtained using the differential scheme (DEM) [4]. We find that these two methods give qualitatively very similar results, and furthermore that the CPA does not predict failure of the cracked material as the crack density is increased (but remains finite), unlike earlier self-consistent schemes. Zero bulk and shear moduli are obtained only in the high density limit. The CPA and DEM are then both used in the Sayers-Kachanov formalism to arrive at estimates of elastic anisotropy for low density cracked systems, and these results in turn are used to obtain Hashin-Shtrikman-type rigorous bounds [5] on the behavior of polycrystalline assemblages of such anisotropic grains. Finally, these results are used to compute compressional, shear, and Rayleigh wave speeds in anisotropic cracked media both with and without saturating pore fluids.

References

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