VORTEX EQUILIBRIA IN CONFINED DOMAINS

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Point vortices and vortex patches are widely used to model flow regions with closed streamlines. The two models can be connected. Point vortices can in fact be considered as vanishing area vortex regions and vortex patches can be obtained as their accretions. The growing process generates a family of regions with closed streamlines.

The literature on this subject is very extensive. For instance, the spectacular result of growing vortex patches that converge in a single Rankine vortex is presented in [1].

The present work is aimed at studying families of regions whose elements are patches with the same circulation as the nascent point vortex region and which are embedded in a potential flow with closed streamlines. Thus, the families here considered are formed by two-level piecewise constant vorticity regions. The vorticity is $\omega = 0$ in the outer part and $\omega = \kappa/A_\omega$ in the inner part, with κ being the circulation of the original point vortex and A_ω the area of the inner patch. The point vortex is the extremum element defined by $A_\omega = 0$. The other extremum is the vortex patch that fills the entire region with closed streamlines.

It can be shown that the vortex patch model has physical relevance in the modelling of finite area separated flow regions. For a proper choice of the jump of the Bernoulli constant, with respect to the external flow, the vortex patch can be considered as the limit solutions of the Navier-Stokes equations for the Reynolds number going to infinity. Thus, the connection between vortex patches and point vortices has practical importance. In fact, if a standing vortex solution does not exist in a flow past a body, it could be conjectured that the entire family of growing vortex patches does not exist and, as a consequence, a finite area separated flow region does not exist either.

For instance, in the flow past a semicircular bump, there is a locus (the Föppl curve) of possible standing single point vortices. In [2] it is shown that, for each standing vortex of this flow, there is a family of vortex patches that goes from the zero area point vortex to a maximum area vortex region that is bounded by the solid body. In [3] the Föppl curve concept has been generalized by showing that a locus of standing vortices can be found in any bounded simply connected domain.

When the solid wall that confines the flow domain has a sharp edge, the flow has to separate at the edge and the number of possible standing vortices reduces to a finite or null number. In [3] it is shown that the existence, or non existence, of standing vortices relevant to flow separating at a wall singularity depends on the nature of the singularity. The present work is aimed at showing, at least for some specific wall geometries, that the non existence of a standing point vortex solution does not allow for the existence of the entire family of vortex patches. This result is in contrast with numerical results available in literature and casts some doubt on their convergence.

References

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