HETEROGENEOUS EQUILIBRIUM CONFIGURATIONS OF N-POINT VORTICES ON THE SPHERE

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This talk will describe a new method of constructing point vortex equilibria on a sphere made up of N vortices with different strengths. Such equilibria, called *heterogeneous equilibria*, are obtained for the five Platonic solid configurations, hence for $N = 4, 6, 8, 12, 20$. The method is based on calculating a basis set for the null space of a matrix obtained by enforcing the necessary and sufficient condition that the mutual distances between each pair of vortices remain constant. By symmetries inherent in the Platonic solid configurations, this matrix is reduced for each case and we call the dimension of the null space the *degree of heterogeneity* of the structure as it represents the number of independent vortex strengths one can use in constructing the equilibrium structure. For the tetrahedron ($N = 4$) and octahedron ($N = 6$), the degree of heterogeneity is 4 and 6 respectively, hence we are free to choose each of the vortex strengths independently. For the cube ($N = 8$), the degree of heterogeneity is 5, for the icosahedron ($N = 12$) it is 7, while for the dodecahedron ($N = 20$) it is 4. Thus, the entire set of equilibria based on the Platonic solid configurations is obtained, including substructures associated with each configuration constructed by taking different linear combinations of the basis elements of the given nullspace.

The method is quite general and with it, one can obtain *all* possible vortex strengths for which a given configuration remains in equilibrium, or *all* possible configurations for which a collection of vortices with given strengths is in equilibrium. The characterization of equilibria by the use of these linear algebraic methods appears to be a new and powerful way of classifying all possible equilibria both on the sphere and in the plane. It also allows one to obtain some 'cheap' and interesting results regarding the probability that N-point vortices placed randomly on a sphere could be an equilibrium for a special choice of vortex strengths. Joint work with M. Jamaloodeen.

References

[1] H. Aref, P.K. Newton, M.A. Stremler, T. Tokieda, D.L. Vainchtein, Vortex crystals, *Adv. Appl. Mech.* **39** 1–79, 2003.

[2] M.I. Jamaloodeen, P.K. Newton, The N-vortex problem on a rotating sphere: II. Heterogeneous equilibria, *under review*, 2006.

[3] P.K. Newton, *The N-Vortex Problem: Analytical Techniques*, Applied Mathematical Sciences Vol. 145, Springer-Verlag, New York, 2001.

[4] P.K. Newton, H. Shokraneh, The N-vortex problem on a rotating sphere: I. Multi-frequency configurations, *in press* Proc. Roy. Soc. Series A, 2005.

[5] P.K. Newton, H. Shokraneh, The N-vortex problem on a rotating sphere: III. Dipoles as interacting billiards, *preprint*, 2006.

Keywords: N-vortex problem; Relative equilibria; Singular value decomposition