INVARIANT DYNAMICAL SYSTEMS IN THE N-VORTEX PROBLEM ON THE SPHERE AND UNSTABLE MOTION OF THE N-RING

TAKASHI SAKAJO*

* Department of Mathematics
University of Hokkaido
Sapporo, Hokkaido 060-0810, JAPAN
sakajo@math.sci.hokudai.ac.jp

We consider the system of the N vortex points with the identical strength on a sphere. As a local effect of the rotation of the sphere, we fix vortex points at the both poles, which are called the pole vortices. When the N vortex points are equally spaced along the line of latitude of the sphere, the configuration is called the "N-ring" or "N-gon". It is a relative fixed configuration[3,4] and its linear and nonlinear stability in the presence of the pole vortices are investigated well[1,2,5]; The stability of the N-ring is determined by the strengths of the pole vortices and the latitude where the N-ring lies.

This talk gives a systematic reduction method of the N vortex problem to invariant dynamical systems based on the linear stability analysis for the N-ring. The reduction is accomplished by making use of the invariant property of the system with respect to the two specific transformations: the shift and the pole reversal transformations, for which the N-ring configuration remains unchanged.

The N-vortex problem is the $2N$-dimensional Hamiltonian dynamical system. We prove that there exists the $2p$-dimensional invariant dynamical system reduced by the p-shift transformation for arbitrary factor p of N. It is also shown that the system can be reduced to the $2[N/2]$-dimensional invariant dynamical system by the pole reversal transformation when the pole vortices are identical, where $[x]$ denotes the maximum integer less than or equals x. The phase space, in which each of the invariant dynamical systems is defined, is a linear space spanned by the eigenvectors obtained in the linear stability analysis and contains the N-ring configuration. Thanks to this linear representation, we can discuss not only the existence of the invariant dynamical systems but also their stability and the inclusion relation between them.

Applying the reduction method, we decompose the system of the large number of vortex points into a collection of invariant reduced systems consists of small vortex points, by which we will describe the unstable evolution of the N-ring, studied in [5,6].

References

[1] S. Boatto and H.E. Cabral, "Nonlinear stability of a latitudinal ring of point-vortices on a nonrotating sphere," *SIAM J. Appl. Math.* **64**, 216–230, 2003.
[2] D.G. Dritschel, "The stability and energetics of co-rotating uniform vortices," *J. Fluid Mech.* **358**, 95–134, 1985.
[3] C. L, J. Montaldi, and M. Roberts, "Relative equilibria of point vortices on the sphere," *Physica D* **148**, 97–135, 2001.
[4] S. Pekarsky and J.E. Marsden, "Point vortices on a sphere: Stability of relative equilibria," *J. Math. Phys.* **39**, 5894–5907, 1998.
[5] T. Sakajo, "Transition of global dynamics of a polygonal vortex ring on a sphere with pole vortices," *Physica D* **196**, 243–264, 2004.
[6] T. Sakajo, "High-dimensional heteroclinic and homoclinc connections in odd point-vortex ring on a sphere," *Nonlinearity* **19**, 75–93, 2005.

Keywords: N-vortex problem, Flows on sphere, Invariant dynamical system