VISCOUS MISCIBLE FINGERING IN SUSPENSION FLOW : STABILITY ANALYSIS IN A RECTANGULAR SLOT

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It is well known that when a suspension of particles is drawn through simple geometries such as tubes and rectangular slots, the suspended particles accumulate behind the advancing meniscus. The phenomenon of shear induced migration causes the particles in the high-shear stress regions near the wall to migrate to the low-shear stress regions near the center of the geometry. Since the fluid streamlines at the center have higher velocities than the fluid streamlines near the walls, there is a net convection of particles towards the meniscus, resulting in the packing of particles at the meniscus. The accumulation phenomenon thus causes a concentration gradient, and therefore a sharp viscosity gradient to be set up at the meniscus, due to the highly nonlinear relationship of viscosity with concentration. Geometries like the rectangular slot and converging parallel plates which have more than one length scale in their cross-sections are susceptible to the viscous miscible fingering phenomenon, being rendered unstable by the unfavorable viscosity contrast.

A simple approach for characterizing the stability of this system is to model it as two immiscible fluids flowing through the rectangular channel, the viscosity of the upstream fluid being a monotonically increasing function of the concentration c of a passive solute. The immiscible interface is impermeable to the solute. A step change in the concentration of the solute is assumed initially in the upstream fluid, thus giving rise to a combination of a miscible interface and an immiscible interface flowing through the rectangular channel in series. The intermediate layer of thickness l between the miscible and immiscible interfaces has a higher concentration of the solute and therefore a greater viscosity. This choice renders the miscible interface unstable due to the unfavorable viscosity combination. The immiscible interface is assumed to have a stable viscosity contrast. The distribution of the solute concentration obeys a simple convective-diffusion equation, the convective velocities being modeled by the usual Darcy-law type relationship with the mobility being a function of concentration. The dispersion relationship (growth rate σ v/s wavenumber k) for this system was determined by employing the Quasi Steady State Approximation (QSSA). For large wavenumbers ($kl >> 1$), the growth rate curve merges into the asymptote corresponding to the analytical result of Tan and Homsy [1]. For sufficiently high wavenumbers, the system is stabilized by diffusion of the solute in the direction tranverse to the flow. For very small wavenumbers ($kl << 1$), the system is stabilized due to the presence of the stable immiscible interface. Thus, there exists a critical thickness l_c of the intermediate layer below which the growth rate is negative for all wavenumbers. This critical thickness l_c increases with increase in Taylor dispersion in the flow direction since this serves to relax concentration and therefore viscosity gradients.

The above simplified model mimics the viscous miscible fingering phenomenon in suspensions well in that it predicts qualitatively the trends of the wavelength and thickness of the accumulated layer at the interface as functions of average concentration. However, it does not incorporate the natural particle enrichment process at the immiscible interface that is observed in suspensions. An effective transport equation for suspension flow in the rectangular slot has been developed to capture this effect. Future work will be directed towards analyzing the stability characteristics of this equation.

References

[1] C.T. Tan and G. M. Homsy, "Stablity of miscible displacements in porous media : Rectilinear flow," *Phys. Fluids.*, **29**(11), 3549-3556, 1986.

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