# CONCENTRATION FLUCTUATIONS IN THE DILUTE SEDIMENTATION OF ANISOTROPIC PARTICLES

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Hydrodynamic interactions in suspensions of anisotropic particles sedimenting under gravity result in a concentration instability, by which particles aggregate into dense clusters surrounded by clarified fluid. While experiments suggest that the instability selects a finite wavenumber corresponding to a typical cluster size in bounded systems [1], a linear stability analysis [2] and periodic simulations [3] both predict a maximum growth rate at zero wavenumber. In the present work we use numerical simulations and theory to investigate the wavenumber selection process of the instability in bounded systems.

First we develop an efficient method based on a point-particle approximation, which allows the simulation of full-scale suspensions of up to 500,000 spheroidal particles in both periodic and slip boundary conditions [4], where the latter are used to qualitatively reproduce the effects of container walls. The salient features of the instability are adequately captured, and the formation of inhomogeneities is shown to be closely linked to the velocity fluctuations in the suspending fluid. In particular, the simulations suggest that the presence of horizontal container walls is necessary for a wavenumber selection to be observed, as they lead to a decay of the initial large-scale velocity fluctuations that otherwise dominate the flow in periodic systems. The sedimentation in finite containers is also characterized by a strong vertical stratification, both at the interface with the clear fluid and inside the bulk of the suspension.

To elucidate the mechanism for the wavenumber selection observed in the simulations, we revisit the linear stability analysis of Koch and Shaqfeh [2] by including the effects of a stable vertical density gradient and of hydrodynamic center-of-mass diffusion. The analysis shows that the growth rates for the concentration fluctuations are damped at low wavenumbers by stratification and at high wavenumbers by center-of-mass diffusion, resulting in a maximum growth rate at a finite wavenumber. A scaling for the dependence of the typical cluster size on the vertical density gradient and on the volume fraction is inferred, and is compared to simulation results in stratified suspensions. In the case of initially homogeneous suspensions, it is argued that the vertical stratification that is observed to develop in the simulations therefore provides a likely mechanism for the wavenumber selection.

**References**

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