A GENERALIZED TOPOLOGICAL SENSITIVITY FOR INVERSE PROBLEMS IN ACOUSTICS

BOJAN B. GUZINA[†•], MARC BONNET[*], AND ALISON MALCOLM[‡]

[†]Department of Civil Engineering, University of Minnesota
[*]Laboratoire de Mécanique des Solides, Ecole Polytechnique, Palaiseau, France
[‡]Institute for Mathematics and its Applications, University of Minnesota
[•]*guzina@wave.ce.umn.edu*

The aim of this study is an extension of the concept of topological derivative as it pertains to the nucleation of infinitesimal inclusions in a reference (i.e. background) acoustic medium. The developments are motivated by the need to aid the solution of inverse scattering problems in terms of a rational initial "guess" about the geometry and material characteristics of a hidden obstacle; an information often required by iterative minimization algorithms. To this end the conventional definition of topological derivative, that quantifies the sensitivity of a cost functional with respect to the creation of an infinitesimal hole, is generalized to permit the nucleation of a dissimilar acoustic medium. On employing the fundamental solution for the background domain, computation of topological sensitivity for the three-dimensional Helmholtz equation is reduced to the solution of a reference, Laplace transmission problem. For generality, it is shown that the sought result can be obtained either via the boundary integral formulation or the Lipmann-Schwinger-type, domain integral representation of the scattered field. Likewise, the developments are presented in a comprehensive setting that permits nucleation of arbitrarily-shaped inclusions (with piecewise smooth boundary) in an infinite, semi-infinite or finite background medium. To cater for engineering applications, explicit formulas are derived for canonical cases when the nucleating inclusion takes spherical or ellipsoidal shape. Through numerical examples it is shown that the featured topological sensitivity can be used, in the context of inverse scattering, as an effective obstacle indicator through an assembly of sampling points where it attains negative values. On varying the material characteristics (namely density) of a nucleating obstacle, it is further demonstrated that the proposed methodology can be used for both geometric and material identification. The presentation will also include a recent generalization of the foregoing (time-harmonic) developments to transient problems.

Keywords: Topological derivative, Helmholtz equation, inverse scattering, transmission problem, acoustic waves, penetrable obstacles, probe method, transient problems.