DISCREPANCY SENSITIVITY FIELD MEASURES AND EXTREMA WITH APPLICATION TO UNCERTAINTY QUANTIFICATION

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Typical problems in design engineering and uncertainty quantification often involve the development of a response surface or response statistics based on a limited number of samples. When these samples are the result of a computationally intensive model, the number of samples is often limited by a finite computational budget. Various approaches have been studied in the literature for “spreading” samples throughout some input (*e.g.*, design variable or random variable input) space, such as regular grid sampling, Latin hypercube sampling [*e.g.*, 1], orthogonal array sampling [2], and Monte Carlo and quasi-Monte Carlo methods [3]. Many of these typical problems, however, are massively multimodal, requiring the samples be “spread” not just over the input space, but over the combined input/input space. Thus, some method is required to quantify how well spread the samples are over some space, and then to use this method to determine new sample points that will (exactly or approximately) optimally contribute to the sample distribution.

The various *discrepancy* definitions [*e.g.*, 3], which probably stem from the work of Kolmogorov [4], are measures of the error between a theoretical distribution and an empirical one based on a set of sample points. The discrepancy can, then, quantify the uniformity of a set of existing sample points but gives no information about where to add new sample points. The sensitivity of a 2-norm discrepancy with respect to the existence of a sample point [5], which quantifies the information provided by *existing* sample points, can be used to determine where new sample points might be well placed, but is only approximate.

This paper shows the derivation of two new measures of the change of the 2-norm discrepancy from adding a new sample point at an arbitrary new location. The first measure, which will be denoted the *discrepancy sensitivity field*, is the linear rate of change of the discrepancy with respect to adding a new sample point with infinitesimal weight. The second measure, denoted the *discrepancy change field*, is the actual change in the discrepancy when a new point is added at an arbitrary point. Thus, both of these measures are indicators of how much “new” information might be provided by the new sample point. Since both of these field quantities are functions of the location of the new sample point, their gradients and Hessian’s can be derived and computed to aid in determining optimal locations for new sample points so as to reduce the discrepancy as much as possible. Several numerical examples are given to show the behavior of these field quantities for different patterns of existing sample points. Ongoing work for demonstrating the uses of these fields for problems in uncertainty quantification is given.

References

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