MECHANICS OF MULTISCALE HIERARCHICAL STRUCTURES

Catalin R. Picu and Monica A. Soare

Department of Mechanical, Aerospace and Nuclear Engineering
Rensselaer Polytechnic Institute
Troy, NY 12180, USA
picuc@rpi.edu

A formulation is proposed for the mechanics of materials with multiscale hierarchical microstructure. The microstructure is geometrically self-similar from scale to scale and forms a sub-domain with Hausdorff dimension smaller than that of the Euclidean space in which it is embedded. Examples of such materials are some porous materials (rocks, aerogels) in which pores with a wide range of dimensions are found, many biological materials (e.g. bone, shell) and some of the newly developed nanostructured hierarchical composites. The microstructure has no characteristic length scale and hence the deformation process has no decoupling scales. In order to address the complexity of such materials that are defined neither in the discrete, nor in the continuum sense, the concepts of stress and strain are re-defined and used to formulate appropriate balance equations. The formulation is based on the concept that the complexity of the geometry must be incorporated in the governing equations rather than being accounted for through the boundary conditions of the problem. Further, the formulation is written in the variational form and a finite element procedure (including new shape functions that account for the complexity of the microstructure) is developed. The solution to several example problems will be presented.

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