DISCONTINUOUS BIFURCATIONS OF ELASTIC-PLASTIC FIBER REINFORCED COMPOSITES IN PLANE STRAIN

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Natural and synthetic fibers have been used to improve the mechanical response of various materials. In this work, the authors quantify the contribution of fibers towards inhibiting the development of discontinuous bifurcations, which are the indicator of localized failure.

A macroscopic tangent stiffness tensor for a fiber-reinforced composite is developed by consistently homogenizing the contribution of fibers in a spherical representative volume element (RVE). A size of RVE is selected so that it is equal to the critical size, which satisfies the following conditions: 1) it is large enough compared to the fiber length, 2) a small increase in the volume beyond the critical size reproduces the volumetric fiber content of RVE, and 3) a small decrease in the volume below the critical size does not reproduce the volumetric fiber content of RVE. Based on these requirements the volumetric fiber content of RVE is found to be an explicit function of the number of fibers in RVE and their aspect ratio. It is assumed that fibers have cylindrical geometry. While the selected distribution of fibers is transversely isotropic in 3D space, its orientation deems it completely isotropic for the purpose of a plane strain analysis. The overall stiffness tensor describes a class of composites that is characterized by elastic fibers, which reinforce the elastic-plastic matrix. Thus, the elastic part of the stiffness tensor contains a mathematical description of the fiber orientation, their aspect ratio and their uniaxial tensile modulus.

The solution for the hardening modulus at the onset of discontinuous bifurcations is obtained from spectral properties of the overall acoustic tensor, which contains the relevant properties of the fibers in addition to the elastic-plastic properties of the matrix. A critical hardening modulus is determined by finding the maximum of the hardening modulus with respect to the unit normal vector that defines the orientation of a singular surface. Solutions are further illustrated on the example of Drucker-Prager yield criterion for a variety of stress states including biaxial compression, uniaxial compression, pure shear, uniaxial tension and biaxial tension. While it is found that the presence of fibers consistently decreases the critical hardening modulus at the onset of yielding, the effect is more pronounced the further away the stress state is from the states of biaxial compression and tension. This is favorable to have since the critical hardening modulus is lower for the latter two stress states. Furthermore, the benefits of fibers are achieved through an increase in the volumetric fiber content. The increase can be accomplished by an increase in the number of fibers and/or by a decrease in the aspect ratio. Additionally, presence of fibers affects the bifurcation direction. The angle between the unit normal and the minor in-plane principal stress increases with an increase in the volumetric fiber content. However, bifurcation directions remain unaffected by the presence of fibers for a specific value of the Poisson ratio, which is equal to 0.25.

In summary, the analytical solutions have demonstrated that fibers consistently inhibit the onset of discontinuous bifurcations. Positive effects of fibers are amplified by increasing their number and/or decreasing their aspect ratio.

References

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