Stationary Oscillation Problem in A Linear Theory of Anti-Plane Elasticity with Microstructure

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The classical theory of elasticity is based on an ideal model of the elastic continuum in which the transfer of loading through any interior surface element occurs only by means of the (force) stress vector. This assumption leads to a description of the deformation of the body in terms of symmetric strain and stress tensors.

Results from analytical models derived on the basis of classical elasticity are in good agreement with experiments performed within the elastic range on numerous structural materials, for example, concrete, steel or aluminium.

The classical theory of elasticity, however, fails to produce acceptable results when the microstructure of the material contributes significantly to the overall deformation of the body, for example, in the case of granular bodies with large molecules (e.g. polymers) or human bones. These cases are becoming increasingly important in the design and manufacture of modern day advanced materials as small-scale effects become paramount in the prediction of the overall mechanical behaviour of these materials.

A dynamic problem of stationary oscillations in the three dimensional elastic micropolar space was formulated by Kupradze. However, a rigorous treatment of the corresponding boundary-value problems in the anti-plane case when stationary waves propagate in an infinite cylinder of any arbitrary cross-section in the direction of the generators of the cylinder or in the unbounded anti-plane space is, to the author's knowledge, still absent from the literature. The main difficulties which arise here are when we try to apply Helmholtz theorem to a solution of the governing equations in the exterior domain. The decomposition is not as straightforward as in the classical elasticity. Only one part satisfies a Helmholtz equation, so that we can impose only one Sommerfeld-type radiation condition.

In this presentation, we introduce time dependency into the theory of anti-plane micropolar elasticity presented by considering the case where all applied forces and hence displacement, strain and stress components are periodic functions of time. Furthermore, we find that uniqueness is guaranteed, provided that the frequency of oscillation is greater than a fixed constant multiple of the speed of longitudinal waves in an infinite micropolar medium.

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