NONLOCAL MODELS BASED ON NONLOCAL DISPLACEMENTS

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We recently introduced the notion of nonlocal displacements as an efficient means to derive nonlocal models. Either an integral approach [1, 2]

$$\text{Average} \qquad \widetilde{\boldsymbol{u}}(\boldsymbol{x}) = \int_{V_x} \alpha(\boldsymbol{x}, \boldsymbol{z})\boldsymbol{u}(\boldsymbol{z})\mathrm{d}\boldsymbol{z} \tag{1}$$

$$\text{Weighting function} \qquad \alpha(\boldsymbol{x}, \boldsymbol{z}) = \alpha(\boldsymbol{x}, r; \ell_\mathrm{c}) \text{ with } r = \|\boldsymbol{x} - \boldsymbol{z}\| \tag{2}$$

or a gradient approach [3]

$$\text{Second-order PDE} \qquad \widetilde{\boldsymbol{u}}(\boldsymbol{x}) - \ell_\mathrm{c}^2 \nabla^2 \widetilde{\boldsymbol{u}}(\boldsymbol{x}) = \boldsymbol{u}(\boldsymbol{x}) \text{ in } \Omega \tag{3}$$

$$\text{Boundary condition} \qquad \widetilde{\boldsymbol{u}} = \boldsymbol{u} \text{ on } \partial\Omega \tag{4}$$

can be used. In Eqs. (1)-(4), $\boldsymbol{u}$ are the (standard) local displacements, $\widetilde{\boldsymbol{u}}$ are the nonlocal displacements and ℓ_c is the intrinsic length. Two-dimensional examples are shown in [4]. Nonlocal displacements can be used to regularize any inelastic softening response, as illustrated with damage and plasticity. This leads to computationally efficient models (especially regarding their consistent linearization) which, at the same time, exhibit a mechanical response (including regularization capabilities) very similar to those of standard nonlocal models.

References

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