ESTIMATING THE BULK MODULUS OF POROUS MATERIALS BASED ON TWO-DIMENSIONAL PORE SPACE IMAGES

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Much theoretical work has been done on the problem of relating the elastic moduli of porous materials, such as rocks and ceramics, to their pore structure [1-3]. These models have shed much light on the mechanical behavior of rocks, and have often been used in an inverse manner, wherein properties of the pore structure are inferred from mechanical or seismic measurements. However, such models have rarely if ever been used in a direct sense, to estimate the moduli based on observation of the pore structure. Part of the problem has been that most theoretical modeling has been based on the assumption that pores can be represented by ellipsoids or spheroids, or special cases thereof, such as spheres, penny-shaped cracks, *etc*. Actual pores in rocks or ceramics are of course more irregular than these idealized shapes. Although non-ellipsoidal three-dimensional pores are not amenable to analytical treatment, two-dimensional pores of essentially arbitrary shape can be analyzed using the complex variable methods developed by Muskhelishvili. However, actual implementation of specific solutions is tedious, and there are as yet only a few solutions available, mainly restricted to pores having symmetrical shapes [4-7]. Based on these results, Zimmerman [4] proposed a scaling law for the pore compressibility, which essentially quantifies the effect of the pore on the overall bulk modulus, in terms of the ratio of perimeter-squared to area.

We have analyzed pore images from some porous ceramics, and from Berea and Fontainebleau sandstone. The image analysis software yields digitized boundary co-ordinates for each pore, as well as the area and perimeter of each pore. The individual pore compressibilities were computed from boundary element calculations, and were also estimated from the aforementioned scaling law. These compressibilities were then averaged, weighted by area, after which both the self-consistent effective medium theory and the differential effective medium theory were used to convert the pore compressibility into a macroscopic bulk modulus. This methodology yielded values of the bulk moduli that were in fairly close agreement with the values measured in the laboratory. Future work will include extension of this approach to the prediction of the shear modulus.

References

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