SPURIOUS MOTIONS ANALYSIS OF TIMOSHENKO'S FLEXURAL WAVES

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Spurious wave motions normally occur in one dimensional finite-element wave solutions, when the flexural wave equations are semi-discritized in space by using non-uniform mesh and then numerically integrated in time. It has been shown that, if a homogeneous elastic bar is discretized into a non-uniform mesh, spurious wave reflections will appear at interfaces of elements with different lengths although no real reflections should occur [1].

As the velocity of flexural waves is frequency dependent, the beam does not have a D'Alembert solution and therefore the solution is dispersive from the outset. On the other hand, as it is difficult to describe the wave motion in the time domain, the spectral analysis of wave motion is adopted. Spectral analysis or frequency domain synthesis is a very efficient approach in analysing waves. Furthermore, spectral analysis is not much a solution technique, but a different insight into the wave mechanics [2]. The main advantage of the spectral approach is that the close connection between waves and vibration becomes apparent.

Wave propagation problems have been solved using lumped mass models [3], as they considerably decrease the amount of operations if an explicit time integration algorithm is used. Recently a new motivation has taken place. The modern third-order implicit integration algorithms have been proposed [3] involving more than one set of implicit equations to be solved at each step. In these cases, if the lumped mass matrix is considered, the amount of operations may become similar to those usual second-order methods.

In previous paper of the author [4] a new lumped mass matrix to solve Timoshenko's flexural wave propagation problems in which the dynamic equilibrium of the moments is taken into account has been developed. The best attributes of this lumped mass matrix may be its low numeric wave number dispersion in comparison with the classical lumped mass matrix. The objective of this paper is to discuss the spurious wave reflections obtained by using this new lumped model.

References

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