SPECTRAL FINITE ELEMENTS FOR STRUCTURAL DYNAMICS ANALYSIS

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The standard finite elements offered in modern commercial codes typically incorporate low-order Lagrangian shape functions with evenly spaced nodes. Unfortunately, standard elements have been found to propagate elastic waves poorly [1]. In this effort we examine the development of finite elements for structural-dynamics applications derived from high-order orthogonal shape functions, which we call *spectral finite elements* (SFE). SFE [2] combine the accuracy of global spectral methods [3] with the geometric flexibility of the finite-element method. Further advantages include the existence of a tensor-product factorization for efficient evaluation of matrix-vector products, and consistently formed diagonal mass matrices. SFE methods have seen extensive, highly successful use in fluid dynamics [2], elastic-wave propagation in solid media for geophysical applications [4], and acoustic-wave propagation [5]. Remarkably, however, we have found no application of SFE to structural components such as beams, plates, and shells. The spectral finite elements discussed here should not be confused with what are known as *spectral elements*, or *modal spectral elements*, in the structural analysis community, which are finite elements based on frequency-dependent shape functions [6, 7].

In this talk, we investigate the use of high-order SFE to solve for the natural frequencies of a clamped-clamped Timoshenko beam. We also calculate its dynamic response to a transverse step-load with centered-difference direct time integration. We compare SFE results with those produced by standard low-order finite elements found in commercial packages. Further, we compare SFE results with their high-order finite-element counterparts with evenly spaced nodes. We find that SFE offer order-of-magnitude improvement in computational accuracy at a given number of model degrees of freedom. Finally, we discuss the formulation and development of SFE for more complex structural components (*e.g.* plates and shells) that provide satisfactory accuracy in production calculations for large models for both small- and large-motion elastic behavior.

References

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