A MODEL OF DISTRIBUTED FAULTING IN CONFINED BRITTLE MATERIALS

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We present a model of distributed damage in brittle materials undergoing triaxial compression. We aim to compute the effective or macroscopic behavior of the material from its elastic and fracture properties; and to predict the microstructures underlying the microscopic behavior [1]. Additionally, we present the numerical implementation of the damage model within a concurrent multiscale framework, and the validation of the model against the experimental data of [2] pertaining to compressive damage in confined ceramics.

Processes of distributed damage in brittle materials have been modelled by a variety of means. Local models of distributed damage have been in some cases misapplied to processes of fracture in brittle solids under tension, where fracture mechanics is expected to govern the behavior of the solid. One of the aims of the present paper is to elucidate the conditions under which damage occurs in a distributed fashion, and therefore can be described by a damage model. We show that distributed damage, as opposed to fracture, is a compressive phenomenon and only occurs when sufficient confinement is present.

The approach followed in the present paper is based on methods of the calculus of variations. We suppose that the displacement field jumps discontinuously across a *singular set* of co-dimension 1, and that the energy is composed of two terms: the elastic strain energy obtained by volume integration outside the singular set; and the cohesive fracture energy obtained by surface integration over the singular set [3]. However, in contrast to recent work on free-discontinuity problems in fracture mechanics, that has emphasized tensile conditions leading to the formation of isolated dominant cracks, here we envision conditions of triaxial compression resulting in a distributed singular set. We specifically consider singular sets that are composed of *recursive or nested faults*, and show that these microstructures or damage patterns suffice to fully relax the energy.

A recursive fault pattern may be constructed by introducing into the solid a family of parallel planar cohesive cracks, or *faults*, and subsequently applying that construction recursively to the intervening matrix between the faults. The state of stress within each level of faulting is uniform, and therefore in equilibrium, although the different levels of faulting are only approximately compatible. Recursive faulting can be implemented simply by means of a recursive call, and the entire microstructure needs not be considered at any time during the construction. The approximate compatibility between levels of faulting has the effect of building additional misfit elastic energy into the microstructure. We estimate this misfit elastic energy simply by modelling the approximate interfaces as rows of dislocation dipoles. This simple estimate permits the calculation of the separation between the faults, and provides a natural termination criterion for the recursive faulting algorithm.

References

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