ISSUES ASSOCIATED WITH MATHEMATICAL MODELING OF HYPER-ANISOTROPIC COMPOSITE MATERIALS

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The development of advanced composite materials brought with it new challenges in their mathematical modeling. A particular example of such materials is the class employing high-temperature thermoplastic polymers reinforced by large volume fractions of fibers. During forming, such fiber reinforced thermoplastic composites (FRTC) exhibit the kinematic constraints of incompressibility and inextensibility in the fiber direction. While most researchers in solid and fluid mechanics are familiar with the former constraint, the consequences of constraining the extensional deformation in one or more directions are not widely understood.

Interest in FRTC has led to several efforts to quantify their behavior. Most applicable to the simulation of forming processes involving thin sheets are the numerical solutions performed assuming a plane stress idealization [1] and those for incompressible plates possessing a direction of inextensibility [2].

This paper first summarizes the past findings of finite element analyses under conditions of plane stress, and presents the results of finite difference analyses of the transverse deformation of plates possessing a direction of inextensibility. In particular, for the case of the two-dimensional finite element analyses, the following observations were made: (1) Unlike the case of incompressible and nearly incompressible media, the non-uniqueness associated with the inextensibility constraint depends not only upon the boundary conditions, but also on the orientation of the material anisotropy (i.e., the fiber orientation); (2) The fiber orientation also influences the number of "spurious" modes associated with an element; (3) When used in the analysis of materials possessing a direction of inextensibility, elements satisfying the Babuška-Brezzi condition for incompressible media appear to be under-constrained.

The governing equations for incompressible plates exhibiting a direction of inextensibility are next reviewed. In the course of the development, the importance of transverse shear response is demonstrated, indicating that lower order plate theories cannot be used to realistically model incompressible/inextensible materials. As is the case with other plate theories, the in-plane and transverse responses uncouple.

References

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[2] P. Simácek and V. N. Kaliakin, "Notes on the Behavior of Transversely Loaded Inextensible Plates," *Int. J. Solids and Structures*, **33**(6), 795-810, 1996.

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