DYNAMIC INSTABILITY OF SHORT MULTI-WALLED CARBON NANOTUBES BASED ON WINKLER AND VAN DER WAALS MODELS

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Vibrational behavior of short carbon nanotubes (CNT's) of aspects ratio between 10 and 50 is of practical significance [1]. In this case, intertube radial displacements of multiwalled CNTs, which are neglected by existing single-beam models [2], could have a significant role. For small aspect ratio the cylindrical shell model is better to describe the behavior of initially co-axial CNTs. Most of the work in the field is devoted to static problems [3].

The parametric vibrations of carbon nanotubes embedded in an elastic matrix under time-dependent axial loading are studied in this paper. Effects of van der Waals interaction forces between the walls of nanotubes are taken into account. Using continuum mechanics an elastic layered shell model is applied to solve the dynamic stability problem of transverse parametric vibrations of carbon cylindrical shells. Axially acting forces of thermal origin are uniformly distributed over the circular shell edges. Both the Gaussian wide-band axial forces and physically realizable forces with known probability distributions are assumed as the tube axial loading. Applying the Donell-Vlasov technical theory of cylindrical shells, the displacements of middle surfaces of the i-th shell in the radial directions are denoted by w_i. Assuming the viscous damping, the linearized van der Walls interaction between shells characterized by the coefficient c and the external shell-matrix interaction with constant d we obtain the coupled system of partial differential equations. The shells are assumed to be simply supported at $x = 0$ and $x = l$ where the transverse displacements and bending moments are equal to zero. As the shell is closed we have also the periodicity conditions for $y = 0$ or $y = 2\Pi r_i$. In order to investigate almost sure or asymptotic dynamic stability of trivial solutions of the derived equations we generate the Liapunov functional as a modification of the full energy of shells, the Winkler foundation and the intershell springs. Stability domains are obtained in an analytical way without discretisation or truncation method. The emphasis is placed on a qualitative analysis of dynamic stability problem. Relations between the coupled shell model and a single shell model with modified stiffness and geometry are analysed. Stability domains are presented in the space of geometric, material and loading parameters.

References

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