EXTENSIONAL FLOWS PAST A VAPOR-LIQUID COMPOUND DROPLET

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Abstract

Extensional flows involving a vapor-liquid hybrid droplet submerged in an immiscible liquid with viscosity $\mu^{(1)}$ in the limit of low-Reynolds number is considered. The multiphase droplet configuration consists of a spherical bubble (a vapor) of radius a partially merged in a sphere of radius b containing a second liquid with viscosity $\mu^{(2)}$. It is assumed that the two spherical surfaces intersect at a contact angle $\frac{\pi}{2}$ for the sake of mathematical convenience and that the surface tension is sufficiently large so that the interfaces have uniform curvature. The study of such problems is motivated by open questions about the flow past merging objects, fusion of biological cells, bubbles attached to drops, and the like. In a constructive and simple manner, steady state solutions involving such a configuration suspended in extensional flows are constructed under creeping flow conditions. The solutions for the corresponding two-dimensional problem for a $2D$ cylindrical multiphase droplet are also obtained. The theory leading to the general expressions for the flow fields under the above mentioned limitations exploit the inverse transformation, and the reflection and translational properties of the biharmonic function associated with the Stokes flow.

The exact solutions for the flow fields are derived in a singularity form suitable for further computations. The salient features of the image singularities (the multipoles/the *Green* functions for the biharmonic equation) are discussed in each case. For the $3D$ vapor-liquid compound droplet, the drag force is computed directly from the singularity solutions. It is observed that the flow fields and the drag force depend significantly on the two nondimensional parameters, namely the viscosity ratio $\Lambda = \mu^{(2)}/(\mu^{(1)} + \mu^{(2)})$ and the radii ratio $\beta = b/a$ of the two-sphere assembly. In particular, it is found that for some extensional flows, there exists a critical value of $\beta = \beta_c$ for each choice of Λ in the interval $0 \leq \Lambda \leq 1$ such that the drag force is negative, zero or positive depending on whether $\beta < \beta_c, \beta = \beta_c$, or $\beta > \beta_c$ respectively. The flow patterns in this case display closed streamlines in the interior of the droplet. In the two-dimensional case, the flow topologies are sketched which show interesting flow patterns, especially in the interior of the cylinder containing a liquid. In particular, a single or a double pair of attached eddies is observed for several values of the radii and viscosity ratios. The size and shapes of the internal eddies vary monotonically with viscosity ratio Λ. Furthermore, the asymptotic analysis leads to a rather surprising conclusion that there is a uniform flow far away from the cylindrical droplet. The exact results provided here may be useful in validating numerical algorithms and codes on multiphase droplet-fluid interactions.

Keywords: compound droplet, low-Reynolds number, drag force, eddies