# A ONE DIMENSIONAL EULERIAN METHOD FOR A VISCOUS DRIPPING FLOW

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Formation of drops via extensional flow and break-off has been much studied (see the review article by Eggers [1]), motivated by a wide range of applications such as ink-jet printing, spinning and drawing of polymer or glass fibres, glass blowing and blow-moulding in the manufacture of containers, light bulbs and glass tubing, rheological measurement by fibre extension and fibre spinning for polymers and glasses. Considerable progress has been made towards the understanding of the breakup of a thin filament into drops, although the exact details of the final stages of breakup are yet to be resolved. However, the evolution of the drop and filament from some initial configuration, and the influence of initial conditions on the final breakup, is still relatively unexplored.

The problem of interest here is a drop of very viscous fluid hanging beneath a solid wall/boundary and extending under gravity, similar to honey dripping from an upturned spoon. The roles of gravity and inertia on the flow were considered in [2, 3], using both one-dimensional and full Navier-Stokes models in a Lagrangian framework. Surface tension was neglected on the basis that a mean diameter $\ell = \sqrt{R_0 L_0}$ of the drop is large compared to the meniscus scale $\sqrt{\gamma/(\rho g)}$, or equivalently that the Bond number $Bo = \rho g \ell^2/\gamma$ is large. Here $g$ is the gravitational acceleration, $\rho$, $\gamma$ are, respectively, the density and surface tension coefficient of the fluid, $R_0$ is a length scale for the drop's cross-section (e.g. the radius of the drop at the wall) and $L_0$ is the initial length of the drop. As the fluid filament extends and gets thinner, this neglect of surface tension may become less justifiable, and an examination of the effect of surface tension is desirable. However, as the filament thins and surface tension potentially becomes important, Lagrangian numerics begin to lose accuracy due to the stretching of computational grids and a decrease in the number of grid elements in the filament region.

In this paper we describe a one-dimensional Eulerian model suitable for analysing the behaviour of viscous fluid drops falling from rest from an upper boundary. The Lagrangian coordinate (a fluid particle label equal to the initial distance $\xi$ from the wall) is sought as a function of time $t$ and that particle's physical distance $x$ from the wall. The move from a Lagrangian to an Eulerian one-dimensional framework results in a formal increase in the order, in both space and time, of the non-linear PDE (for $\xi = Z(x,t)$) that must be solved. In addition we move from a problem in a fixed computational domain $0 < \xi < L_0$ to a moving boundary problem in the domain $0 < x < L(t)$ where the actual drop length $L(t)$ must be determined as part of the problem. Nevertheless, the method allows examination of development and behaviour of the flow from early time, when a drop and filament begin to form, out to large times when the bulk of the fluid forms a drop at the bottom of a long thin filament which connects it with the upper boundary, and hence enables a better examination of the thin fluid filament.

## References

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