ERROR MEASURES FOR FINITE DIFFERENCE SOLUTIONS

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The finite difference method was largely replaced by the finite element method in the late 1950's and 1960's for the approximate solution of solid mechanics and structural applications. This was due to the perception that the finite element method could represent a wider range of complex geometries and boundary conditions than the finite difference method. Recent developments have allowed the finite difference method to represent practically any problem that can be represented by the finite element method [1].

A resurgence of the finite difference method as a prime analysis tool in structural analysis and solid mechanics can be anticipated because of two characteristics. Firstly, the finite difference method does not have the "overhead" of learning that is associated with the finite element method because it is directly related to the governing differential equations and both types of boundary conditions. Secondly, the finite difference method is easier to program because of the simplicity of the finite difference approximations of the derivatives.

Although the finite difference method has the advantages just outlined, there is one area where the finite difference method lags behind the finite element method. There are few, if any, point-wise discretization error measures available to evaluate the accuracy of a specific finite difference model. Such measures have been developed for the finite element method [1] but they cannot be applied to finite difference models because the point-wise discretization error is estimated for the finite element model by computing the residuals associated with the point-wise solution of the governing differential equations.

This work identifies metrics that can serve as point-wise discretization error measures for finite difference results [2]. These point-wise refinement guides can be used to adaptively refine the finite difference models until results of adequate accuracy are achieved. The error measures developed here are also applicable the finite element results because of their universal nature.

The overall approach taken for developing these point-wise error measures is to indirectly relate the approximate solution to some characteristic of the exact solution. Four point-wise metrics are developed using the following three relationships between the continuous and discrete results: 1) the satisfaction of the compatibility equations, 2) the satisfaction of a secondary set of equilibrium equations and 3) the requirements of strain continuity.

Each of the four point-wise error measures is demonstrated on a simple two-dimensional shear panel problem and the convergence is examined. The most promising error measures are applied on a more complex problem. This second problem is used to identify the best point-wise refinement guide.

References

[1] John O. Dow, *A Unified Approach to the Finite Element Method and Error Analysis Procedures*, Academic Press, New York, 1999.
[2] Amy Kay Siegfried, *Point-Wise Discretization Error Measures for Finite Difference Solutions*, Master's Degree Thesis, University of Colorado, 2005.

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