THE MECHANICAL RESPONSE OF HETEROGENEOUS STRUCTURES BY A FUNCTIONAL PERTURBATION METHOD

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Except from specific cases, linear differential equations with non-constant coefficients have no analytical solutions. This type of equations governs the behavior of heterogeneous media and/or problems formulated in curvilinear coordinates. Currently, approximate solutions are obtained by numerical (FEM), energy (Rayleigh-Ritz) or by Galerkin type of methods. Perturbation, multiple scale and homogenization methods have been also developed for cases when the non-uniformity of the coefficients is small or cell-like.

Functional (as opposed to parametric) treatment of heterogeneity has been partially exploited in studies related to effective properties, i.e., when grain size, or correlation lengths, are very small compared to the global dimensions. Formal use of the Fréchet series as a fundamental tool in perturbation methods is still not common.

Therefore, the main objective of this study is to propose a generic Functional Perturbation formulation, which analytically and systematically solves any set of linear differential equations, representing nonuniform, integrable (either random or not) material properties, of finite or infinite media to any desired degree of accuracy. The formulation generalizes the Functional Perturbation Method (FPM) which was recently developed for the analysis of heterogeneous structures, e.g. [1-4]. The method is based on considering the unknown function as a functional of morphology. Then, writing the governing differential equations in a convolution-like format, and using the properties of the Dirac operator (and its derivatives), a generic formulation for the solution of any set of linear non-homogeneous differential equations is obtained. The method is not limited for cases where the non-uniformity is small, and is appropriate for any morphology either stochastic or not.

The method and its important features are demonstrated for both the conductivity and elasticity problems. By using the proposed Functional Perturbation Method, effective properties are obtained directly from the original differential equations, without the use of projection operators or transformation to integral equations. Also, Based on multiple point convolutions, useful measures of non-uniformity such as stress field fluctuations, which are essential for reliability and material design, can be calculated. Formally, the FPM can be applied to finite bodies of any size and geometry without substantial modifications.

***Keywords**: Random, morphology, convergence.*

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