A DISSIPATIVE MODEL FOR THE VISCOPLASTICITY OF DRY AND FLUID-SATURATED GRANULAR MEDIA

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This paper revisits the "purely dissipative" model proposed several years ago [1] as a general continuum model for the history-dependent viscoplasticity of non-colloidal particle dispersions. Essential to the model is a positive-definite fourth-rank viscosity tensor $\boldsymbol{\eta}$ depending on the history of deformation. In the reduced form considered here, $\boldsymbol{\eta}$ is an isotropic function of a history-dependent 2nd-rank "texture" or "fabric" tensor $\mathbf{A}$, which gives stress as a tensor-valued function of fabric and strain-rate tensors. This paper considers several special cases appropriate to systems ranging from Stokesian suspensions to dry granular media.

For Stokesian suspensions, a formula for $\boldsymbol{\eta}(\mathbf{A})$ given by the analogous theory of linear elasticity, together with a corotational memory integral for $\mathbf{A}$, provides a compelling model of transient viscosity and normal stresss evolution in simple shear [5, 3]. However, one extremely rapid mode of relaxation is required to mimic the incomplete reversal of stress on abrupt reversal of shearing. This suggests that non-hydrodynamic effects are implicated, and it establishes a kinship to liquid-saturated granular media with sustained particle contact.

In the case of granular media, the isotropic version of the above model reduces to the Reiner-Rivlin form proposed previously [2], which encompasses a quasi-linear model proposed recently for dense rapid granular flow [4].

Since isotropic models cannot represent the effect of flow-induced anisotropy on yield surfaces and viscometric normal stresses, attention is given here to a more general forms, involving nonlinear dependence on both fabric and strain rate. Two important time scales are highlighted, a grain-inertia time scale for dry granular media [2, 4], and a viscous-frictional time scale that appears to be implicated in recent experiments on completely saturated granular media.

References

[1] J. D. Goddard. Dissipative materials as models of thixotropy and plasticity. *Journal of Non-Newtonian Fluid Mechanics*, 14:141–60, 1984.

[2] J. D. Goddard. Dissipative materials as constitutive models for granular materials. *Acta Mechanica*, 63:3–13, 1996.

[3] J.D. Goddard. A dissipative anisotropic fluid model for non-colloidal particle dispersions. *submitted for publication*, 2005.

[4] P. Jop, Y. Forterre, and O. Pouliquen. Crucial role of sidewalls in granular surface flows: consequences for the rheology. *Journal of Fluid Mechanics*, 541:167–92, 2005.

[5] V. G. Kolli, E. J. Pollauf, and F. Gadala-Maria. Transient normal stress response in a concentrated suspension of spherical particles. *Journal of Rheology*, 46:321–34, 2002.

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