REDUCED MODELS FOR CHAOTIC DYNAMICS OF A FLUID-CONVEYING PIPE

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The nonlinear dynamics and stability of fluid-conveying pipes has been established as a generic paradigm in the modern theory of dynamical systems [1]. This simple system permits both theoretical and experimental investigation in parallel. A fluid-conveying pipe describes a fluid-structure interaction system, perhaps in the simplest form. Simple physical modifications to this elementary system allow one to study the effect of an elastic foundation, additional spring supports or lumped masses, without expending significant effort, in both theoretical and experimental framework. These variants of the elementary system exhibit a kaleidoscope of dynamical behaviours. This system also represents a more general class of problems involving momentum transport, such as travelling strings, bands and chain-saws.

In the present study, a high-dimensional model for chaotic vibration of a cantilever conveying fluid having an end-mass is investigated. The nonlinear partial differential equation describing the oscillations of the pipe is converted into a finite set of coupled nonlinear ordinary differential equation (ODEs) using a Galerkin projection with the uniform cantilever-beam modes as basis. Depending on the parameter range of interest, it turns out that the order of the coupled set of ODEs is much larger (up to 18 degree-of-freedom) in order to obtain a convergent solution. In order to construct a bifurcation diagram with the non-dimensional flow velocity as unfolding parameter, a prohibitive amount of computational effort is necessary in a single-processor (serial) personal computer. To alleviate this computational limitation, such bifurcation diagrams for high-order chaotic system are constructed in parallel computer simulation using multi-threading (i.e. auto-parallelism). A significant computational gain is achieved in terms of time-saving through parallel-processing. The computational gain permits a reliable construction of bifurcation diagrams in the chaotic regime within a reasonable time frame.

Subsequently, the efficacy of a reduced-order model [2,3] based on proper orthogonal mode (POD) is contrasted with the high-dimensional model (with uniform cantilever-beam mode as basis). Numerical results demonstrate the merits and drawbacks of the POD-based bifurcation diagram, depending on the non-dimensional flow-velocity chosen to construct the POD.

References

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