NORMAL STRESSES AT STAGNTION POINTS OF DROPS, BUBBLES AND SOLIDS IN VISCOUS AND VISCOELASTIC FLUIDS

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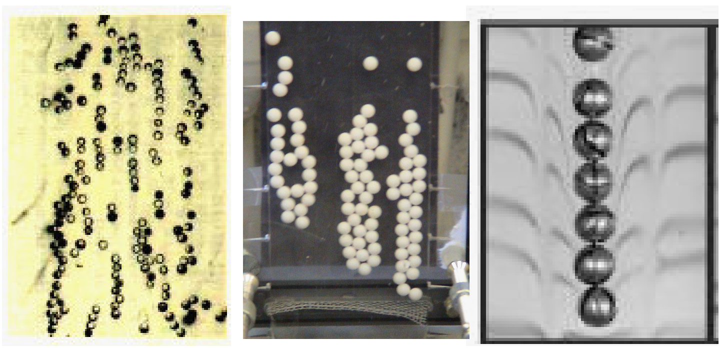
The robust tendency of small particles from nanometers to centimeters to chain in all kinds of motions must be associated with a powerful and local feature of particle-fluid interactions. We argue that the chaining of bubbles, drops and solid spheres (figure 1) is a consequence of the same dynamics that controls the orientation of long bodies moving relative to the stream, across the stream in Newtonian fluids and along the stream in viscoelastic fluids. This dynamics is mainly controlled by a reversal of the normal stress at a point of stagnation



(a) (b) (c)

Figure 1. Flow induced microstructure. Spheres line up in the direction of flow (a) Extensional flow, (60-70 μm spheres) (b) fluidization (3 cm spheres) and (c) sedimentation (3 cm spheres) in a 1% aqueous PEO solution.

A point of stagnation on a stationary body in potential flow is a point at the end of a dividing streamline at which the velocity vanishes. In a viscous fluid all the points on the boundary of a body have a zero velocity but the dividing streamline can be found and it marks the place of zero stress near which the velocity is small.

Wang and Joseph (2004) have presented an analysis of potential flow of a second order fluid over an ellipse and elliptical bubble using classical irrotational airfoil theory framed in terms of functions of a complex variable. The effect of the viscoelastic terms is opposite to that of inertia; the normal stress at a point of stagnation can change from compression to tension. This causes long bodies to turn into the stream and causes spherical bodies to chain. The viscoelastic stresses extend the rear end of a rising bubble, tending to the cusped trailing edge observed in experiments. The same stresses cause small gas bubbles to chain in viscoelastic fluids and disperse in Newtonian fluids.

The stagnation points of a sphere in a uniform stream are [r = a, θ = 0 or π] where, respectively

$$T_{rr} + p_\infty = \left[\frac{9(2\alpha_2 - \alpha_1)}{a^2} - \rho\right]\frac{U^2}{2} \mp \frac{6\eta U}{a}.$$

The viscous contribution gives rise to compression $-6\eta U/a$ at the front stagnation point and to tension $6\eta U/a$ at the rear. The sign of the stress due to inertia and viscoelasticity is the same at [θ = 0, π] and it is a tension when $9(2\alpha_2 - \alpha_1) > \rho a^2$. The quantity $2\alpha_2 - \alpha_1$ is strongly positive; for example, for the liquid M1 (Hu et al. 1990), α_2 = 5.39 and α_1 = -3 (g/cm). Hence, if a^2 is not too large the stress at the stagnation points is a tension, reversing the compression due to inertia. This tension has a profound effect on the microstructure and is an ingredient in the glue that holds chained spheres together and turn long bodies into the stream. These viscoelastic effects are stronger for smaller particles (proportional to $1/a^2$) than for larger ones. The chains of small particles exhibited here may be a realization of this prediction. How small is really small? Do nanoparticles chain?

J. Wang, D.D. Joseph, 2003 Potential flow of second order fluid over a sphere or an ellipse. *J. Fluid Mech.*, **511**, 201-215

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